

Heterogeneous Beliefs and Business Cycles

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discussion by:

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- Very interesting and challenging theoretical investigation
- The aim is to give a chance to beliefs to be a driver of the business cycle:
 - labor is hired in advance of a shock realization (investment)
 - Epstein-Zin preferences (large EIS)
- When beliefs are extrapolative:
 - asset and labor markets become more volatile
- Intuition for the role of **heterogeneous beliefs**

2-period economy: competitive equilibrium (homogeneous beliefs)

- two period economy with discount β : $t = 0, 1$
- two agents (A, B) with endowments: $y_t^A = \{1, 0\}$ (**worker**); $y_t^B = \{0, (1 - s, 1 + s)\}$ (**firm**)
- states: $s^t = \{s_0, (s_{1l}, s_{1h})\}$ with $prob(s_1 = s_{1h}|s_0) \equiv \pi_1(s^1 = s_{1h}) = \pi_{1h}$
- log utility and homogeneous beliefs: $u(c) = \log c$ and $\pi_{1h} = \pi_{1h}^A = \pi_{1h}^B$

- An Arrow-Debreu competitive equilibrium implies:

$$q_1^0(s^t) = \beta \pi_1(s^t) / y_1(s^t) \Rightarrow \begin{cases} q_1^0(s_{1l}) = \frac{\beta \pi_{1l}}{1-s} \\ q_1^0(s_{1h}) = \frac{\beta \pi_{1h}}{1+s} \end{cases}$$
$$c_0^A(s^0) = \frac{1}{1+\beta} \quad \text{and} \quad \begin{cases} c_1^A(s_{1l}) = \frac{1-s}{1+\beta} \\ c_1^A(s_{1h}) = \frac{1+s}{1+\beta} \end{cases}$$

Market with a bond and equity share (homogeneous beliefs)

- Market with a risk-free bond ($b^A + b^B = 0$) and shares over B endowment ($e^A + e^B = 1$)
- Budget constraint for A and prices:

$$\begin{cases} c_0^A + p_b b^A + p_e e^A = y_0^A, & t = 0 \\ c_1^A(s^1) = b^A + e^A y_1^B(s^1), & t = 1 \end{cases}$$

$$p_b = \sum_{s^1} q_1^0(s^1) = \beta \frac{1 - s(\pi_{1h} - \pi_{1l})}{1 - s^2}$$

$$p_e = \sum_{s^1} q_1^0(s^1) y_1^B(s^1) = \beta$$

- Same (AD) allocation with $b^A = 0$ and $e^A = 1/(1 + \beta)$. Moreover:

$$E(R^e(s^1)/R^b) = (1 - s^2(\pi_{1h} - \pi_{1l})^2) / (1 - s^2) \quad \text{(equity-premium)}$$

$$\eta_1^A(s_{1h}) = \eta_1^A(s_{1l}) = 1/(1 + \beta) \quad \text{(wealth share)}$$

2-period economy: competitive equilibrium (heterogeneous beliefs)

- Beliefs are heterogeneous with: $\pi_{1h}^A = \gamma\pi_{1h} \neq \pi_{1h}^B = \pi_{1h}$ and $\pi_{1l}^A = \phi\pi_{1l}$ (with $\phi = \pi_{1l}^A/\pi_{1l}$)
- An Arrow-Debreu competitive equilibrium implies:

$$\ddot{q}_1^0(s^t) = \frac{\beta}{1+\beta} (\pi_1^A(s^t) + \beta\pi_1^B(s^t)) / y_1(s^t) \Rightarrow \begin{cases} \ddot{q}_1^0(s_{1l}) = \left(\frac{\beta\pi_{1l}}{1-s}\right) \left(\frac{\phi+\beta}{1+\beta}\right) \\ \ddot{q}_1^0(s_{1h}) = \left(\frac{\beta\pi_{1h}}{1+s}\right) \left(\frac{\gamma+\beta}{1+\beta}\right) \end{cases}$$
$$\ddot{c}_0^A(s^0) = \frac{1}{1+\beta} \quad \text{and} \quad \begin{cases} \ddot{c}_1^A(s_{1l}) = \frac{1-s}{\phi+\beta} \phi \\ \ddot{c}_1^A(s_{1h}) = \frac{1+s}{\gamma+\beta} \gamma \end{cases}$$

- If $\gamma > 1$ (optimistic worker A), $\ddot{c}_1^A(s_{1h}) > c_1^A(s_{1h})$ and $\ddot{q}_1^0(s_{1h}) > q_1^0(s_{1h}) \rightarrow$ less risk sharing in $t = 1$ since agents agree that the price of shifting consumption from the low to the high increases!

Market with a bond and equity share (heterogeneous beliefs)

- Price of the bond and equity are (let $\gamma > 1$ and $\phi < 1 \rightarrow$ optimistic worker A):

$$\begin{aligned}\ddot{p}_b &= \sum_{s^1} \ddot{q}_1^0(s^1) = \frac{\beta}{1+\beta} \left(\frac{\pi_{1l}(\phi + \beta)}{1-s} + \frac{\pi_{1h}(\gamma + \beta)}{1+s} \right) < p_b \\ \ddot{p}_e &= \sum_{s^1} \ddot{q}_1^0(s^1) y_1^B(s^1) = \beta = p_e\end{aligned}$$

- Same (AD) allocation with $\ddot{b}^A < 0 = b^A$ and $\ddot{e}^A > e^A$. Moreover:

$$E \left(\ddot{R}^e(s^1) / \ddot{R}^b \right) < E \left(R^e(s^1) / R^b \right) \quad (\text{equity-premium})$$

$$\ddot{\eta}_1^A(s^1) = \begin{cases} \phi / (\phi + \beta) < \eta_1^A(s^1), & s^1 = s_{1l} \\ \gamma(\gamma + \beta) > \eta_1^A(s^1), & s^1 = s_{1h} \end{cases} \quad (\text{wealth share})$$

- Optimistic beliefs \Rightarrow **increases** the interest rate and **decreases** the risk premium (**proposition 2**); **increases** wealth share under the good shock, **decreases** under the bad shock (**proposition 4**)

Comments - simple examples to build up intuition

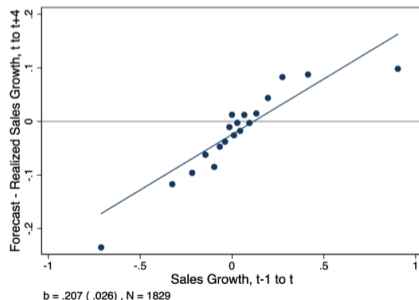
- The paper could benefit from simple examples to build up intuition (maybe in the appendix)
- Example: two or three period models or **periodic endowment/productivity** economies as in chapter 8 of [Ljungqvist and Sargent \(2012\)](#)
- Isolate effects of belief heterogeneity and labor being hired in advance of shock realization

- Model shows that extrapolative beliefs can amplify the effect of productivity shocks
- Those type of beliefs can also lead to larger amplitudes of **credit/leverage cycles** in an economy → potentially relevant for policymakers
- Misperceptions (**unawareness**) about future productivity may lead to excessive debt which the agents cannot repay → can we think about default waves ([Prescott and Townsend, 1984](#); [Modica et al., 1998](#))?

Comments - data sources on beliefs

- The authors introduce a **taxonomy of beliefs** that may have different implications for the business cycle (extrapolative, intrapolative, etc.)
- Using the *Atlanta Fed/Stanford Survey of Business Uncertainty*, [Barrero \(2022\)](#) presents evidence that managers → are not overoptimistic; are overprecise; and **over-extrapolate**

(c) Forecast Errors vs. Past Sales Growth



Open questions...

- 1 Excellent model that captures typical narratives that waves of pessimism/optimism drive the business cycle. Can we use the model to think about **wages of leverage/credit**? **Efficiency** of policy interventions?
- 2 The model depends on a state variable that captures the distribution of assets across agents. Is this a limitation to **add further heterogeneity**?
- 3 Which **data** sources can be used to discipline the type of beliefs used in the model?

References

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Only if necessary

- use if necessary